Fall 2013 Interstellar Problems

- 51. Let α be an irrational number, j an integer, and $i = \sqrt{-1}$. How many different values of j is $e^{i2\pi j\alpha}$ equal to 1?
 - (A) none (B) one (C) more than one
 - (D) an infinite number (E) varies depending on α
- 52. In a single elimination basketball tournament, a team is eliminated when it loses one game. The champion is the last remaining team with no losses. In a tournament with n teams, how many games must be played in order to determine a champion?

(A)
$$\frac{n!}{2!(n-2)!}$$

(B) 2^{n-1}

(D)
$$\sum_{k=1}^{\ell} \frac{n}{2^k}$$
, where $n-1 < 2^{\ell} \le n$

- (E) Not enough information about the tournament is given.
- 53. In a group of 100 students 32 are taking AP Calculus, 29 are taking AP History, and 28 are taking AP Statistics. In addition, 12 are taking both Calculus and Statistics, 11 are taking History and Statistics, and 9 are taking Calculus and History. Further 5 students are taking all three subjects. How many students are not taking any AP classes?
- 54. The function r(x) is a cubic (3rd degree) polynomial. The function p(x) is a quadric (4th degree) polynomial. Given the value in the chart below, find the coefficient of x^4 in p(x).

x	1	2	3	4	5
r(x)	4	3	6	7	0
p(x)	4	3	6	7	-36

(A)
$$-\frac{7}{2}$$
 (B) $-\frac{5}{2}$ (C) $-\frac{3}{2}$ (D) $-\frac{1}{2}$ (E) $\frac{1}{2}$

55. The real number x is in the closed interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ and is a solution to the equation $\log_{10}(8\sin x) + \log_{10}(5\cos x) = 1$. What is the exact value of x?

(A)
$$\frac{\pi}{4}$$
 (B) $\frac{\pi}{3}$ (C) $\frac{3\pi}{8}$ (D) $\frac{5\pi}{12}$ (E) $\frac{\pi}{2}$

56. Solve for integer n:

$$\frac{3^n}{n^3} = 27.$$

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58. Two integer numbers, x and y, are randomly chosen between 1 and 2011, inclusive. Which is closest the probability that |x - y| < 3?

(A) 0.000025 (B) 0.00025 (C) 0.0025 (D) 0.025 (E) 0.25

59. In a new card game with two players, each player has a deck of 25 cards numbered from 1 to 25. Each player randomly selects a card from his deck. The numbers on their cards are multiplied together and the first player pays the second player the number of pennies in the product. For this to be a fair game, how many pennies should the second player pay the first player for the privilege of playing this game?

(A) 0 (B) 1/625 (C) 1 (D) 169 (E) 625

60. Let AC = 5, CD = 3, and AD = 4. Points E and B are on \overline{AD} and \overline{AC} , respectively, so that the area of $\triangle ABE$ is half of $\triangle ACD$, and $EB \perp AC$. What is AB?

(A) 2 (B) $1 + \sqrt{2}$ (C) $\frac{5}{2}$ (D) $2\sqrt{2}$ (E) 4

61. If $x + 2x + 3x + \dots + 2012x = 2013$, what is $2013x + 2014x + 2015x + \dots + 4024x$?

(A) 6037 (B) 6038 (C) 6039 (D) 6040 (E) 6041